BY CHAND T. JOHN, PhD

WHAT VALUE COULD FRACTALS ADD TO BIOMEDICAL IMAGE ANALYSIS?

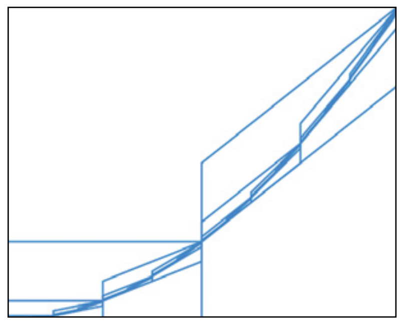
We collect large amounts of biomedical image data, hoping to glean insights into our biological world. While deep learning has become popular for finding features that, for example, distinguish between benign and malignant tumors in biomedical images, how these features relate to conclusions we care about remains a mystery hidden in a labyrinth of neural networks.

Fractals, "self-similar" shapes whose parts resemble the overall shape itself, better connect biomedically relevant properties to their internal parameters. A fractal has properties like fractal dimension, a measure of how its complexity changes across scales, and lacunarity, a measure of sparsity and non-uniformity, that relate to patterns of interest in biomedical images. For example, fractal models of the lung have revealed correlations between tumor presence, higher fractal dimension, and lower lacunarity. Fractal modeling of brain cancers has allowed determination of tumor stages using fractal dimension. These medically relevant properties of a fractal are closely tied to its self-similar structure, which itself emerges from the fractal's defining parameters. In this way, fractals offer a tantalizing strength in connecting biomedically relevant properties back to their internal mathematical parameters in a way that deep-learning models currently do not.

While fractals lend insight into their own internal structure, it's tempting to ask whether fractal models can go even deeper. Could a fractal's parameters somehow relate to other meaningful aspects of an image's geometry, similar to how Newton's laws relate geometric parameters of planetary trajectories to the geometry of underlying gravitational force fields?

One type of fractal offers a surprisingly tangible set of parameters: smooth curves that resemble the fractal's own shape. Smooth fractals are curves such as parabolas, which, as it turns out, are fractals themselves. For example, a pair of mappings called affine maps, carefully chosen, can squeeze and stretch a box repeatedly in a way that causes the resulting shape to converge toward a parabola.

Starting with multiple smooth fractals, combining their affine maps into one aggregate mapping, and repeatedly applying that aggregate mapping to a box, results in a shape that, despite its complexity, is completely characterized by those smooth curves with which we started.



Starting with a square and repeatedly applying two affine transformations leads to a set of parallelograms that converge to the graph of $y = x^2$, a polynomial curve. Courtesy of Chand John.

Could smooth curves comprising such fractals be chosen based on meaningful geometric aspects of an image, such as vector fields of most rapid changes in pixel intensity, thereby tying biomedically relevant properties of fractals back to other geometric features of biomedical images? While more research is needed to decide whether fractals, alongside deep learning, can join the collection of mathematical gems that propel biomedical image analysis to new heights, it's an intriguing idea to explore. □

The polynomial curves on the left fully represent the fractal on the right. Courtesy of Chand John.

DETAILS

Chand John, PhD, is a senior software engineer and educator.