BY BIN DONG

The Implicit Representation of Biological Shapes and Forms

Imaging, geometric modeling, representation and computing of shapes and forms are important components of modern computational biology. These processes apply across wide spectra of scales, genotypes and phenotypes, from microarray imaging for genomics, to neuroimaging of human brains. One of the most fundamental image processing problems is the representation of shapes and forms. There are two popular ways of representing biological shapes: parameterization-based (explicit) representation and implicit representation. Parameterization-based representation codes important shape information into geometric variables (such as the vertices of a triangle in a triangular mesh, and how the vertices relate to one another—i.e., whether they are connected by an edge) and topological variables (such as whether there are holes in the shape or not, e.g. the difference between a ball and a donut). In contrast, implicit representations are frequently described via level set functions and their siblings. A level set function is usually defined to take negative values inside the shape and positive values outside, and hence its zero level set (i.e., the set of points on which the function takes zero values) describes the shape. Both types of representations have their own advantages and disadvantages.

The major advantage of using triangular meshes to represent biological shapes is the efficiency of data storage and algorithmic development. We can represent and process a high-resolution high-accuracy shape without using excessive physical memory. However, one drawback of using a triangulated surface is its inflexibility in terms of topological changes (e.g., merging of two bubbles). This is rather critical for some cases. Topological changes affect many shape-processing procedures, e.g., shape restoration and segmentation. Whenever topological alterations occur, the original triangular mesh becomes degenerate and demands retriangulation or surface correction. Take shape restoration, for example: Topological changes may happen so often that it demands constant shape retriangulation, which makes processing algorithms computationally inefficient.

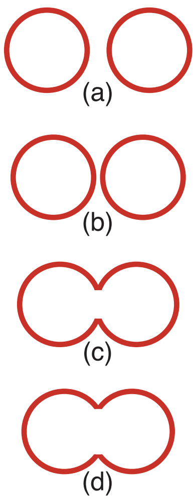

Figure 1. Topological changes induced by merging two bubbles.

As for implicit representations of shapes, taking level set functions as an example, the major advantage is their flexibility in terms of topological changes. Whenever shape-processing introduces topological changes, implicit representations are more flexible and convenient than parameterization-based representations. Let us look at the simple example in Fig. 1, where we are animating the merging of two bubbles in 2-D. From (a) to (d) in Fig. 1, the two bubbles are growing at a constant speed with their centers fixed. Topological change happens at (c), where the two bubbles touch and then merge into one bubble. It is very easy to implement this motion when the bubbles are represented by a level set function. All we need to do is solve a certain differential equation. If the bubbles are parameterized, however, we would need to constantly check if merging is about to happen, and when it does, reparameterize the shape. This makes the computational implementation rather complicated.

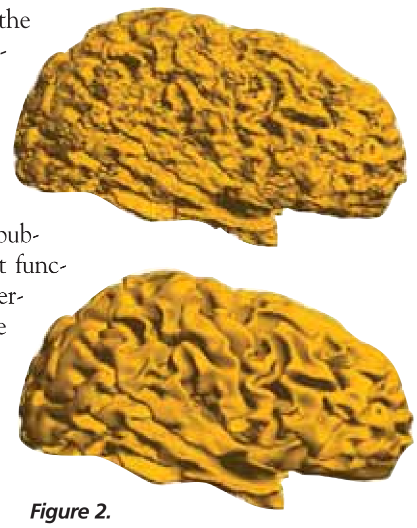

Figure 2. top: original, noisy cortex data; below: NLM processed cortex.

In addition to topological flexibility, implicit representations are more natural in representing biological shapes for practical purposes, because the shapes extracted from modern imaging data, e.g., MRI, CT and ultrasound images, are intrinsically implicitly represented in the first place. Also, since level set functions are usually defined on standard Euclidean grids, most level-set based algorithms are very easy to implement. However, in contrast to the parameterization-based representations, implicit representations are usually not very efficient in storing the data. Indeed, surface data, which is essentially 2-D, is implicitly saved as a 3-D function. The problem becomes more severe when we are dealing with high-resolution shapes. In general, efficient storage and manipulation of high-resolution implicit shapes is challenging when we need to keep all the existing features of the representation.

One example where the level set representation may be more appropriate is cortical surface restoration, where topological changes are unavoidable. In Fig. 2, we show how nonlocal means (NLM) [1], where the cortex is represented by a level set function, can automatically remove the many small isolated bubbles that arise from segmentation errors.

REFERENCES:

[1]. Bin Dong, Jian Ye, Stanley Osher, Ivo Dinov. Level-set-based nonlocal surface restoration. In *Multiscale Modeling and Simulation*, 7(2), 589-598, 2008. □

DETAILS

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