BY CHIH-WEN KAN AND MIA K. MARKEY, PhD

Mutual Information

Mutual information (MI) is defined in information theory as a measure of the dependencies between two random variables. There are many biomedical applications in which it is beneficial to quantify the information content using a measure such as MI. In classification problems, MI is used as a dependence measure to select features such that they are dissimilar from each other in order to reduce feature redundancy. MI can also be used in database retrieval. The MI is calculated between a query item and every entry in the database in order to identify the entry in the database that is most similar to the query item.

In image processing, it is also used extensively as a similarity measure for image registration and for combining multiple images to build 3D models. We will use the application domain of medical image registration to illustrate the utility of MI.

The mutual information of random variables A and B is defined as

$$I(A,B) = \sum_{a,b} p(a,b) \log \left(\frac{p(a,b)}{p(a)p(b)} \right)$$

where $p(a,b)$ is the joint probability distribution function of A and B, and $p(a)$ and $p(b)$ are the marginal probability distribution functions of A and B, respectively.

Thus, in the context of medical image registration, MI measures the distance between the joint distributions of the images' gray values $p(a,b)$ and the distribution when the two images are independent from each other. It is a measure of the dependence between the two images. Since the mutual information $I(A,B)$ is the reduction in the uncertainty of A due to the knowledge of B, when $p(a) = p(b)$, the uncertainty is minimal and the reduction of uncertainty is maximized.

In medical imaging, it is often necessary to compare images of a patient that are acquired at different times or by different modalities. For example, images may be taken pre- and post-operatively in order to assess the successfulness of a surgery. To facilitate the interpretation of such sets of images, registration—the process of aligning multiple images—is necessary. The goal of registration is to identify a transformation that maps each point in one image to the corresponding point in the other image.

One approach to image registration is based on defining landmarks or fiducial points in the images. By determining how to align those landmarks, one can determine how to transform one image to match the other. However, manual definition of landmarks is time consuming, may be difficult even for an experienced observer, and suffers from intra- and inter-reader variability.

Another approach to image registration is to determine a transformation based on a measure of the similarity of the images, such as MI. Since larger MI corresponds to more similarity of the two images, MI is maximized in registration algorithms.

In image registration, the goal is to determine a transformation of one image such that the MI between the transformed image and the reference image is maximized. Different types of transformations may be considered based on the application. The simplest class of transformations only permits rotations and translations. In medical imaging, a wider variety of scaling and shape changes are often needed, including non-linear transformations that allow for non-uniform changes across the image. An optimization algorithm is applied to dynamically search among transformations for the one with maximal MI.

In image registration, the goal is to determine a transformation of one image such that the mutual information between the transformed image and the reference image is maximized.

MI has been shown to be especially valuable for registering multi-modality images. For example, computed tomography (CT), positron emission tomography (PET), and magnetic resonance imaging (MRI) images of the same patient provide complementary information. Registration based on MI enables a healthcare provider to directly correlate the data from such different imaging techniques. MI has also shown promise for registering time series images. A series of images over time is often used to evaluate tissue function in addition to structure. □

DETAILS

Chih-Wen Kan is a graduate student in The University of Texas Department of Biomedical Engineering. She works on developing diagnostic decision support systems in Dr. Mia Markey's Biomedical Informatics Lab (http://bmil.bme.utexas.edu/).